

THE NEW INTERPRETATION OF QUANTUM MECHANICS AND HIDDEN PARAMETERS^{1,2}

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ABSTRACT. The new interpretation of quantum mechanics is based on a complex probability theory. An interpretation postulate specifies events which can be observed and it follows that the complex probability of such event is, in fact, a real positive number. The two-slit experiment, the mathematical formulation of the complex probability theory, the density matrix, Born's law and a possibility of hidden variables are discussed.

1. INTRODUCTION

In this paper the old problem of Quantum Mechanics (QM) – the interference of probability amplitudes – is investigated from the point of view of a complex theory of probability (C-TP). It can be said that the "mystery of QM" is not explained until now, if we understand under the term "to explain" something deeper than "to say how to calculate observed numbers".

Here we suggest a way to imagine what is going to happen in the two-slit experiment; the calculated numbers, however, are the same as before.

¹A part of this paper was presented at the Conference "Hadron Structure '76" held in Smolenice (Czechoslovakia) 15.-19.11.1976. A revised form can be found in [14].

²This paper is an identical copy of preprint No. NBI-HE-81-4 of Niels Bohr Institute (Copenhagen) from November 1981. It has never been published regularly.

Preparation of this paper was supported by the Grant No. RN 19982003014 of the Ministry of Education.

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QM is necessarily a probabilistic theory, but the probability is used here in a completely non-classical way [1]. The role of probability is played by the probability amplitude, but this amplitude is complex whereas the probability must be real.

It is well recognized that it is virtually impossible to ascribe a definite trajectory to the electron, but it is equally impossible to ascribe a set of trajectories having different probabilities to it; in this respect the electron differs from a Brownian particle. It is possible to assign the Feynman probability amplitude to each trajectory, to add them, and at an appropriate moment to take the squared modulus using the RPI (rule of principal indistinguishability): for principally indistinguishable alternatives add the amplitude, for distinguishable ones add the probabilities [1], [2].

For an electron we suggest the following concept of the trajectory: it is a couple $(\gamma_+, \gamma_-) = \gamma$ of two possible trajectories, where γ_+ and γ_- are oriented forward and backward in time, respectively. To each such "trajectory" there corresponds a complex probability Φ given by

$$(1) \quad \Phi(\gamma) = \phi(\gamma_+)[\phi(\gamma_-)]^*,$$

where $\phi(\gamma_+)$ is the Feynman amplitude for the path γ_+ . Equation (1) in a way resembles the definition of a density matrix $\rho(x', x) = \psi(x')\psi^*(x)$. It can be said that the electron moves independently in both directions of time with generally different trajectories.

Now, the two-slit experiment (Exp.1) can be simply explained. There are four possible ways to pass from s to x through slits 1 and 2:

- (i) γ_+ and γ_- go through 1,
- (ii) γ_+ and γ_- go through 2,
- (iii) γ_+ goes through 1 and γ_- goes through 2,
- (iv) γ_+ goes through 2 and γ_- goes through 1.

Diagrammatically it is written as (Fig.1).

Note that this means decomposition of the event $(s \rightarrow x)_{\text{through } 1+2}$ into four disjunctive subevents (!); really no numbers enter Fig.1.

Let us suppose that the observation of an electron at a point P means that both γ_+ and γ_- pass through P (so called *interpretation postulate*). Let us consider the second experiment (Exp.2) in which the electron is observed at slit 2. From the interpretation postulate

it follows that the possibilities (iii) and (iv) are excluded and we have (Fig. 2). Thus the C-probability of an event $(s \rightarrow x)_{\text{through } 1+2}$ is given by

$$\begin{aligned}\Phi_{Exp. 1}(x) &= \left(\sum_{\gamma \in (i)} + \cdots + \sum_{\gamma \in (iv)} \right) \Phi(\gamma) = \\ &= |\langle x|s \rangle_1|^2 + |\langle x|s \rangle_2|^2 + 2Re\langle x|s \rangle_1 \langle x|s \rangle_2^* \\ \Phi_{Exp. 2}(x) &= \left(\sum_{\gamma \in (i)} + \sum_{\gamma \in (ii)} \right) \Phi(\gamma) = |\langle x|s \rangle_1|^2 + |\langle x|s \rangle_2|^2,\end{aligned}$$

because, for example

$$\sum_{\gamma \in (iii)} = \left(\sum_{\gamma_+ \ni 1} \phi(\gamma_+) \right) \left(\sum_{\gamma_- \ni 2} \phi(\gamma_-) \right)^* = \langle x|s \rangle_1 \langle x|s \rangle_2^*.$$

The cases (iii) and (iv) may be called *the interference events*. In our theory the difference between Exp. 1 and Exp. 2 does not arise because of a disturbance of the electron by the measuring apparatus, but in our two experiments we observe truly different events.

The interpretation postulate thus replaces RPI; IP is a natural element of probabilistic description of Nature: this is the rule determining which "event" in the theory corresponds to the event observed in an experiment.

The correspondence between the classical particle going through the point x and its mathematical description by a curve containing x is considered as self-evident. The correspondence between an electron passing through x and its description by $\gamma = (\gamma_+, \gamma_-)$ with $x \in \gamma_+ \cap \gamma_-$ may be considered as strange, but we think this strangeness is apparent, induced by our classical background.

The concept of $\gamma = (\gamma_+, \gamma_-)$ contains a certain element of non-locality; this is exactly what is observed in QM – see the Einstein-Rosen-Podolski paradox [3]. For example, the question "how can the electron passing through slit 1 know whether the other slit is closed or open?" can be answered simply: the electron may find out this using its backward trajectory γ_- (supposing γ_+ passes through slit 1). If both slits are open, the electron can use possibilities (iii) and (iv); these possibilities cannot apply if one or other of the slits is closed. The interference character (i.e. so-called wave properties) of electrons are natural in C-TP; the usual R-TP is rather of an "adding" character.

2. COMPLEX THEORY OF PROBABILITY

Brownian motion can be mathematically described [4] by the R-TP (Ω, Σ, P) , where

$$\begin{aligned}\Omega &= \text{space of elementary events} = \\ &= \{\text{all possible trajectories of a Brownian particle}\} = \\ &= \left\{ \gamma : \mathbb{R} \rightarrow \mathbb{R}^4 \mid \gamma(\tau) = [t(\gamma(\tau)), \vec{x}(\gamma(\tau))], \frac{d}{d\tau}t(\gamma(\tau)) > 0 \right\}, \\ \Sigma &= \text{system of events} - \text{system of measurable subsets of } \Omega, \\ P &= \text{non-negative } \sigma\text{-additive measure on } \Sigma, \quad P(\Omega) = 1.\end{aligned}$$

We shall use the symbolic notation $P(\gamma)$ (exact for Ω finite). The probability of the transition $[x_1 \rightarrow x_2]$, $x_1, x_2 \in \mathbb{R}^4$ is then given by

$$P[x_1 \rightarrow x_2] = \sum_{\gamma(0)=x_1, \gamma(1)=x_2} P(\gamma),$$

where $P(\gamma)$ is the Wiener measure on Σ .

Now we shall give the exact definition of the notions introduced in the Introduction.

Definition. The *trajectory* of quantum particle is a couple $\gamma = (\gamma_+, \gamma_-)$ of two oriented curves in space-time, $\gamma_+, \gamma_- : \mathbb{R} \rightarrow \mathbb{R}^4$, such that $\frac{d}{d\tau}t(\gamma_+(\tau)) > 0$, $\frac{d}{d\tau}t(\gamma_-(\tau)) < 0$. (In this paper we consider the case of non-relativistic QM.) *C-TP* is the system (Ω, Σ, Φ) , where $\Omega = \{\gamma = (\gamma_+, \gamma_-)\}$, Σ = system of "measurable" subsets of Ω and Φ is a complex function defined on Ω satisfying axioms (A1)–(A4) given below.

The *trajectory* γ^+ *adjoint to* $\gamma = (\gamma_+, \gamma_-)$ is defined by $\gamma^+ = (\check{\gamma}_-, \check{\gamma}_+)$, where $\check{\gamma}_\pm(\tau) = \gamma_\pm(-\tau)$; $E^+ = \{\gamma^+ \mid \gamma \in E\}$ for $E \in \Sigma$ (and $E^+ \in \Sigma$ is supposed).

The event $E \in \Sigma$ is called *hermitian* ($E \in \Sigma_{herm}$) if $E^+ = E$.

Let us denote $\Omega_+ = \{\gamma_+ \mid (\gamma_+, \gamma_-) \in \Omega\}$, $\Omega_- = \{\gamma_-\}$. The event $E \in \Sigma$ is called *pure* ($E \in \Sigma_{pure}$), if $E = A \times \check{A}$, where $A \subset \Omega_+$, $\check{A} = \{\check{\gamma} \mid \gamma \in A\}$ and $A \times \check{A}$ means the Cartesian product.

The event $E \in \Sigma$ is called *mixed* ($E \in \Sigma_{mix}$) if E is a disjoint union of pure events.

Φ is assumed to satisfy the following axioms.

- (A1) Φ is a σ -additive complex measure on Σ , i.e., symbolically,
 $\Phi(E) = \sum_{\gamma \in E} \Phi(\gamma)$, $E \in \Sigma$,
- (A2) $\Phi(\gamma^+) = [\Phi(\gamma)]^*$, i.e. $\Phi(E^+) = [\Phi(E)]^*$,
- (A3) $\Phi(\gamma_+, \gamma_-) = \Phi(\gamma_+ \times \Omega_-) \cdot \Phi(\Omega_+ \times \gamma_-)$ or, more generally,
 $\Phi(A \times B) = \Phi(A \times \Omega_-) \cdot \Phi(\Omega_+ \times B)$.

Thus it suffices to know Φ for events of type $\gamma_+ \times \Omega_-$. This axiom implies the statistical independence of γ_+ and γ_- . Moreover, in QM the following rules hold:

- (A4) $\Phi(\gamma_+ \times \Omega_-) = \phi(\gamma_+) = e^{iS(\gamma_+)} = \text{Feynman's amplitude for } \gamma_+$.

If $E = A \times \check{A} \in \Sigma_{pure}$, using (A3), (A2) and (A4) we obtain

$$\begin{aligned} \Phi(E) &= \Phi(A \times \Omega_-) \cdot \Phi(\Omega_+ \times \check{A}) = \\ &= \Phi(A \times \Omega_-) \cdot [\Phi(A \times \Omega_-)]^* = \left| \sum_{\gamma_+ \in A} \phi(\gamma_+) \right|^2. \end{aligned}$$

Thus the C-probability of pure events is (after appropriate normalization) equal to the usual real probability. The same is true for mixed events, because $\Phi(E) = \sum \Phi(E_k)$ if $E = \bigcup_{disj} E_k$, $E_k \in \Sigma_{pure}$. From (A3) we have (for $A = \Omega_+$, $E = \Omega_-$): $\Phi(\Omega) = |\Phi(\Omega)|^2$ and thus $\Phi(\Omega) = 1$ (assuming $\Phi(\Omega) \neq 0$).

Interpretation postulate (IP).

Let us suppose that the experiment is prepared in such a way that the presence of the electron at (space-time) points x_1, \dots, x_n , y_1, \dots, y_m is measured. Let us consider an event in which the presence of an electron at x_1, \dots, x_n was confirmed and at y_1, \dots, y_m excluded. This situation is described in our theory as an event

$$\begin{aligned} E_{x,y} &= E_{x_1, \dots, x_n, y_1, \dots, y_m} := \\ &= \{(\gamma_+, \gamma_-) \mid x_1, \dots, x_n \in \gamma_+(\mathbb{R}) \cap \gamma_-(\mathbb{R}), y_1, \dots, y_m \in \gamma_+(\mathbb{R}) \cup \gamma_-(\mathbb{R})\}. \end{aligned}$$

We can assume that all observations and preparations of quantum systems may be described in terms of $E_{x,y}$. We see that $E_{x,y} \in \Sigma_{pure}$ and

$$\Phi(E_{x,y}) = \left| \sum_{\substack{\gamma_+ \ni x_1, \dots, x_n, \\ \gamma_+ \not\ni y_1, \dots, y_m}} \phi(\gamma_+) \right|^2.$$

Generally, the observable events are of the type Σ_{mix} while the dynamics is given in terms of $\phi(\gamma_+) = \Phi(\gamma_+ \times \Omega_-)$ alone. This is the reflection of the fact that in QM the observed quantities (= probabilities) are expressed bilinearly in dynamical quantities (amplitudes or wave functions).

3. THE DENSITY MATRIX

Our discussion of the two-slit experiment was simplified because the experiment was described in space variables only. Here the transition to the space-time description $x = (t, \vec{x})$, $S_1 = (t_1, \vec{x}_1)$ etc. will be assumed. Let us suppose that in the n -slit experiment the electron is measured at slits S_1, \dots, S_m , ($0 \leq m \leq n$). The event observed is then given by

$$E(x) = \left(\bigcup_{k=1}^m E_{kk} \right) \cup \left(\bigcup_{k,l=m+1}^n E_{kl} \right),$$

$$E_{kl} = \{(\gamma_+, \gamma_-) \mid s, x \in \gamma_+(\mathbb{R}) \cap \gamma_-(\mathbb{R}), S_k \in \gamma_+(\mathbb{R}), S_l \in \gamma_-(\mathbb{R})\}$$

and its probability by

$$\begin{aligned} \Phi(E(x)) &= \sum_{k=1}^m |\phi_k|^2 + \sum_{k,l=m+1}^n \phi_k \phi_l^*, \\ \phi_k &= \sum_{x,s,S_k \in \gamma_+(\mathbb{R})} \phi(\gamma_+). \end{aligned}$$

Let us suppose that the electron was observed at x_0 at the moment t_0 , then it has passed through a slit system with some slits equipped with measuring apparatus. This is so-called *preparation* of the electron. The corresponding event

$$E = \bigcup_{k=1}^n A_k \times \check{A}_k \in \Sigma_{mix}$$

is defined as the set of all "trajectories" $\gamma = (\gamma_+, \gamma_-)$ passing through slits and satisfying IP at space-time points where electron was measured. Let us set

$$\rho(\vec{x}_+, \vec{x}_-; t) = \sum_{\substack{\gamma \in E \\ \gamma_{\pm}(1) = (t, \vec{x}_{\pm})}} \Phi(\gamma_+, \gamma_-) = \sum_{k=1}^n \psi_k(\vec{x}_+, t) \psi_k^*(\vec{x}_-, t),$$

where

$$\psi_k(\vec{x}_+, t) = \sum_{\substack{\gamma \in A_k \\ \gamma_+(1) = (t, \vec{x}_+)}} \phi(\gamma_+).$$

ψ_k may be considered as a (non-normalized) wave function and ρ as the (non-normalized) density matrix. Because the ψ_k 's are not normalized, we have means for constructing general mixed states. For example, in Exp. 2, the state of an electron behind the slits is described by $\sum_{k=1}^2 \psi_k(\vec{x}_+, t) \psi_k^*(\vec{x}_-, t)$, where ψ_k is the wave function of an electron passing through the k -th slit ($k = 1, 2$).

The interference character of C-TP may be seen clearly from the following property of C-TP. In C-TP, there are events with $\Phi(E) = 0$ having subevents $E_1 \subset E$ with $\Phi(E_1) \neq 0$ (in R-TP: $P(E) = 0$, $E_1 \subset E \Rightarrow P(E_1) = 0$). This is exactly what is observed in QM: $\Phi(E(x))$ may be zero for some $x \in \mathbb{R}$ but $\Phi(E_{kk}(x)) > 0$ and $E_{kk}(x) \subset E(x)$. Thus, the null-events E in C-TP are such that $\Phi(E_1) = 0$ for each $E_1 \subset E$.

4. THEORY OF PROBABILITY AND BORN'S LAW

Let us stress that the Theory of Probability (Ω, Σ, P) is a theoretical scheme, unverifiable directly, especially the quantity $P(E)$ is not generally measurable, because this would be the measurement of the probability of an individual event and this is absurd. Only the relative frequency $p(E)$ of an "independently repeated identical" event may be measured and only for such an event does the application of the Law of large numbers (LLN) give the relation $p(E) = P(E)$ [4].

The aim of the Theory of Probability (TP) can be formulated as follows: to find a theoretical construction (using generally unverifiable concepts) which enables us to calculate the relative frequencies of collective events. From this point of view a certain similarity between the intuitive notion of probability and properties of P is rather accidental and irrelevant. This means that P , for example, need not be a real positive function, but the relative frequency obtained by LLN must. We conjecture that both R-TP and C-TP (including IP) form a priori possible bases of a description of the real world. The quantum mechanical experience shows undoubtedly that C-TP is a true

fundamental TP. R-TP may be then considered as a classical approximation to C-TP in the limit in which the interference effects (or non-real C-probabilities) are negligible.

Let us now consider the LLN in C-TP. Let $E = E(x_0)$ be an arrival of the electron at the point $x_0 \in \mathbb{R}$ on a screen. The complementary event to E is $F = \bigcup_{x \neq x_0} E(x)$ and $E, F \in \Sigma_{mix}$.

The C-probabilities $\Phi(E)$, $\Phi(F)$ are positive, but must be normalized, because we are interested only in electrons measured somewhere on the screen; so let us set $\tilde{\Phi}(E) = Z^{-1}\Phi(E)$, $\tilde{\Phi}(F) = Z^{-1}\Phi(F)$, $Z = \Phi(E \cup F)$. Let E_k, F_k , $k = 1, 2, \dots$ be repetitions of the events E and F . We shall assume that the usual formula $\Phi(E_k \cap F_l) = \Phi(E_k) \cdot \Phi(F_l)$ holds for independent events. This formula holds for $\tilde{\Phi}$ as well. Using the standard argument [4] from the derivation of LLN we obtain $p(E) = \Phi(E)$. It follows from the fact that Φ of an event $[|p(E) - \Phi(E)| \geq \varepsilon > 0]$ tends to zero and that the events with $\Phi(E) = 0$ do not happen (this basic assumption is not usually stated explicitly). In terms of usual QM (if $E(x) \in \Sigma_{pure}$ for simplicity) we have

$$\Phi(E(x)) = \left| \sum_{\gamma_+(1)=x} \phi(\gamma_+) \right|^2 = |\psi(x)|^2,$$

where the wave function ψ is not normalized. Thus, the Born's law will hold for the normalized wave function

$$|\tilde{\psi}(x_0)|^2 = \tilde{\Phi}(E(x_0)) = p(E(x_0)).$$

We think that the relative frequency p should be used in the formulation of Born's law instead of P . It is interesting that in our C-probabilitistic approach, this law is not a postulate but a consequence.

5. INTERPRETATION POSTULATE – A DISCUSSION

We shall show that it suffices to assume the validity of IP for macroscopic systems only. This follows from the hereditary property of IP: if IP holds for a system, then IP holds for its subsystems.

Let us consider Exp.2 from Chapter 1. We shall suppose that the electron going in the forward/backward time-direction interacts

only with photons going in the same time-direction. This agrees with the fact that quantum laws are written in amplitudes ϕ . The interference event (iii) now looks as follows (Fig. 3), where the gauge 1/0 detects the scattered/not scattered photon. Using IP for the system {electron + photon}, we see that the event described above is not observable. It follows that the event described above is not observable. It follows that the event (iii) in Exp. 2 (considering the electron as the system and a photon as a measuring apparatus) cannot happen for the system {electron} and this is exactly the assertion of IP for the case of {electron}.

6. STRAIGHTFORWARD PHYSICAL INTERPRETATION AND THE C-BROWNIAN MOTION

Another interpretation of C-TP may be given which is mathematically equivalent but physically deeper than the description given above.

Let us suppose (in the case of non-relativistic QM) that there are two sorts of electrons – the forward and backward ones with respect to the time-direction of their evolution and that the electron of different sorts move independently. To the event when a forward/backward electron moves along the path γ there corresponds the complex probability $\phi(\gamma) = \text{ampl. for } \gamma / \phi^*(\gamma)$. The Interpretation Postulate must be reformulated.

IP': To the observation of an electron at a space-time point x there corresponds the following event E_x . E_x is the event, when there was simultaneously the forward electron moving along γ_+ and the backward one moving along γ_- such that both γ_+ and γ_- pass through x . The formula

$$\phi(E_x) = \sum_{\gamma_+, \gamma_- \in E_x} \phi(\gamma_+) \phi^*(\gamma_-)$$

holds because of independence of the forward and backward electrons.

Thus, for example, the physical transition $x \rightarrow y$ is interpreted as an event of a transition $x \rightarrow y$ of a forward electron together with the transition $y \rightarrow x$ of some backward electron.

This interpretation allows us to consider the Feynman amplitude $\phi(\gamma)$ directly as the complex probability neglecting the fact that this

is a complex number, because the amplitude $\Sigma\phi(\gamma)$ always will be positive for an observable event defined by IP'.

The motion of a quantum mechanical particle may be considered as the complex analogue of a Brownian motion (this idea has already been suggested but in the context of R-TP [5]-[11]). Let us proceed further with this idea: the quantum particle can be viewed as an object scattered by some medium (in the sense of C-TP). Let us define the subquantum particle as the "particle" unscattered by this medium and we expect it be a "point-like complex object". The subquantum particle moves in the "complex" medium and its motion is influenced by a large number of independent random effects, each of them being very small. The "complex" medium should be interpreted as a subquantum vacuum. These questions will be considered in detail in the next paper; let us mention that the analogy between the propagator of the Schrödinger equation and the propagator of the Brownian particle and the analogy between the Schrödinger and Fokker-Planck equation has already been hinted in [12], [13], but our concept allows us to consider the Schrödinger equation *as* the Fokker-Planck equation for the "complex" Brownian particle.

Let us discuss briefly possible consequences of this approach in order to find a possible deterministic interpretation of QM.

- (i) The probability amplitude (or wave function) is interpreted as the C-probability, quantum superposition as an addition of C-probabilities, and the principle of indeterminacy as an analogue of the indeterminacy of Brownian motion.
- (ii) The hidden parameters may be defined as parameters describing the subquantum vacuum. These parameters are principally unobservable (the positions and velocities of atoms of gas are also unobservable) and this causes the statistical character of QM. Theorems on non-existence of hidden parameters [3] do not work here, because these parameters are described by complex statistics.
- (iii) So-called quantum jumps appearing at a measurement can be understood in "complex" statistics purely statistically. The act of measurement is the act of finding which C-probable possibility was realized, analogously to the case when the position of a Brownian particle is measured.
- (iv) It can be expected that the system {particle + vacuum} is deterministic and that the indeterministic character of QM

arises from our neglecting the parameters that describe the subquantum vacuum. There is also another indeterminacy, because we can observe only certain events described in IP'. The motion of a subquantum particle is "C-Brownian-deterministic" in one direction of the time (i.e. the description by $\phi(\gamma_+)$). On the other hand, the physical transition incorporates both directions of time (i.e. $\phi(\gamma_+)\phi^*(\gamma_-)$) which are mixed together.

7. CONCLUSIONS

It follows from Bell's inequality and from the experiments described in [3] that the local hidden parameter theory cannot explain the observed facts, providing that this theory is based on the R-theory of probability. In the C-TP Bell's inequality does not hold (since it is based on positivity of probability). Thus the local hidden parameter theory using C-TP instead of R-TP may solve the above dilemma.

On the other hand, the C-TP brings a deeper and more direct understanding of the phenomenon of quantum interference. The main goal of the paper was to generalize the notion of probability in such a way to include quantum mechanics in a natural and direct manner.

ACKNOWLEDGEMENTS

Sincere thanks are to my brother, Dr. V. Souček (Charles University, Prague), who proposed the physical interpretation of C-TP in terms of forward and backward electrons. The autor thanks Niels Bohr Institute for their hospitality during his stay in Copenhagen. I also thank Eva Murtinová for her help in typing of this manuscript.

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